MATHEMATICAL CHALLENGE

CHRISTMAS - 2016

Problem 1.
Which numbers between 1 and 20 can be written as the sum of the decimal digits in the square of some natural number?
Find a natural number such that the sum of the digits in its square is 2017. Is there such a number for 2018? Justify your answer.

Problem 2.
Find a function $f(x)$ defined for all real x such that the rotation by $\pi/2$ about the origin maps its graph into itself.

Problem 3.
The Sheriff of Nottingham wants to play the following game with Robin Hood using a sheaf of arrows. The first to play has to split the sheaf into two non-empty piles, then the players have to in turn split every pile that has more than one arrow. The person who makes the last move is the winner.
The Sheriff insists on starting the game, but allows Robin Hood to choose any initial number of arrows in the sheaf between 25 and 100.
Which of these numbers can Robin choose to ensure victory? What is the winning strategy? Justify your answer.
The same question for the number of arrows between n and $4n$ for any n.

Remarks.

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be handed in on or before Friday 27 January 2017 to either Dr. H. Ahmadinezhad (SCH.1.13) or Prof. A.P. Veselov (SCH.1.02), who will be the judges for the Challenge.