Loughborough University

Department of Mathematical Sciences

MATHEMATICAL CHALLENGE

CHRISTMAS - 2019

Problem 1. Define the sequence $(x_n) = (1,\, 1,\, 2020,\, 4082419,\, 8250566779, \ldots)$ by the recurrence

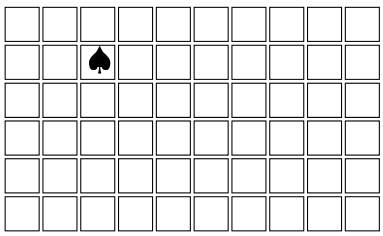
$$x_{n+1}x_{n-1} = x_n^2 + 2019, \qquad n \in \mathbb{N},$$

with the initial terms $x_0 = x_1 = 1$.

Prove that x_n are integers for all natural n and find $\lim_{n\to\infty} x_{n+1}/x_n$.

Problem 2. Let $\{m_1, \ldots, m_n\}$ be a set of pairwise coprime natural numbers. Prove that there are infinitely many natural numbers k such that the shifted numbers $k + m_1, \ldots, k + m_n$ are also pairwise coprime.
Show that the same conclusion is true for the set $\{2, 4, 6, 7\}$ and describe all sets with this property that: a) have 4 elements; b) have 2019 elements.

Problem 3. Anna and Elsa play a game with a pack of Arandelle playing cards (which have 60 cards) arranged face-up in a rectangle as shown below. The position of the Ace of Spades is indicated:



At each turn one of them makes the rectangle smaller by removing any number of rows or any number of columns from the edge of the rectangle. Who finally has to take the Ace of Spades loses the game.
Anna takes the first turn. Can she win the game, and if so what should be the winning strategy? Justify your answer.

Remarks.

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be handed in on or before January 31, 2020 to either Prof. A.P. Veselov (SCH.1.02) or Dr. B. Winn (SCH.M.05), who will be the judges for the Challenge.