**Loughborough University**

**Department of Mathematical Sciences**

# MATHEMATICAL CHALLENGE

## CHRISTMAS - 2023

**Problem 1.** Andy believes that a year is good if he can split its decimal digits into groups with equal sums. For example, the year 2024 is good, but both 2023 and 2025 are clearly not.
When will it happen next that one good year is followed immediately by another good year?

**Problem 2**. Let $A$ be an $n \times n$ real matrix with positive matrix elements, and $B = A^{-1}$ be its inverse. Prove that $B$ cannot have more than $n^2 - 2n$ zero matrix elements.
For each of $n = 3$ and $n = 4$, find an example of an $n \times n$ matrix with positive matrix elements and with inverse having exactly $n^2 - 2n$ zero matrix elements. Do such matrices exist for arbitrary $n > 2$? Justify your answer.

**Problem 3**. Scrooge, being in a good mood, is offering Bob Cratchit a chance to get an extra bonus for Christmas. On a piece of paper Scrooge wrote all 26 letters of the English alphabet in a secret order. The deal is that Cratchit can try to recover this order by asking which letters are written on any particular set of places of his choice. Scrooge will then provide only the total set of letters without revealing their precise positions.
Cratchit has only 5 attempts before he must guess the secret sequence. Can he get the bonus? Would he be able to do it if he would only be allowed 4 attempts? Justify your answer.

*Remarks.*

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be scanned and e-mailed on or before January 31, 2024 to either Prof. A.P. Veselov (`A.P.Veselov@lboro.ac.uk`) or Dr. B. Winn (`B.Winn@lboro.ac.uk`), who will be the judges for the Challenge.